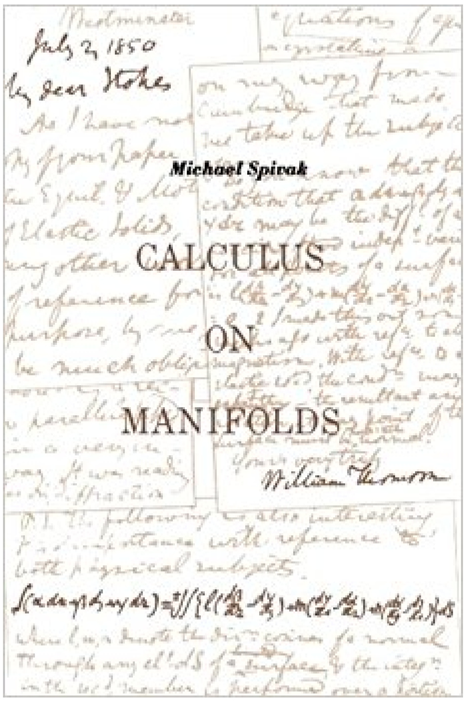
Calculus On Manifolds: A Modern Approach To Classical Theorems Of Advanced Calculus



Synopsis

This little book is especially concerned with those portions of Ã¢â ¬Â•advanced calculusÃ¢â ¬Â• in which the subtlety of the concepts and methods makes rigor difficult to attain at an elementary level. The approach taken here uses elementary versions of modern methods found in sophisticated mathematics. The formal prerequisites include only a term of linear algebra, a nodding acquaintance with the notation of set theory, and a respectable first-year calculus course (one which at least mentions the least upper bound (sup) and greatest lower bound (inf) of a set of real numbers). Beyond this a certain (perhaps latent) rapport with abstract mathematics will be found almost essential.

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Customer Reviews

A supplementary text for undergraduate courses in the calculus of variations which provides an introduction to modern techniques in the field based on measure theoretic geometry. Varifold geometry is presented through and appraisal of Plateau’s problem.

With its instantly recognizable cover, Calculus on Manifolds (1965) is a classic and for cultural reasons, every serious math/theoretical physics grad student should have read this deceptively slim volume by his/her first year of grad school. As one of the earliest texts on this subject, it’s a admirable and ambitious attempt to modernize multivariable calculus (and the author wrote it at the age of 25!), but for several reasons, the text is of limited pedagogical value.The most serious problem with this text is the number of errors. While they do not necessarily disrupt the overall picture the author is trying to convey, a few statements in the book are wrong in a substantial way, and many others require additional hypotheses or slight modifications. The astute student should carefully consider each statement the author makes before assuming its validity, and if false, spend some time thinking about how to make the statement true. Reading this book with this degree of scrutiny is, of course, an exceedingly time-consuming challenge, but could be very rewarding....Compounding the frequent occurrence of errors is the terseness of the text. Nonobvious implications and arguments in proofs are frequently glossed over by words like "obviously" or "clearly" or merely strung together by a series of equal signs.Problems with this book in detail:1) a) The notation is poorly chosen, especially with respect to (alternating) forms and tangent vectors and spaces, such as the use of $\Lambda^k(V)$ instead of the more standard $\mathcal^k(V)$ or $\Lambda^k(V^*) for the space of alternating tensors on V. In the addendum, he corrects this usage but claims that $\Omega^k(V)$ has become the standard notation for this space. However, $\Omega^k(M)$ is actually more commonly used as a notation for the vector space of differential k-forms on a manifold M.b) Sometimes, even minor typos will result in considerable confusion on the part of the student. For instance, on p. 97, Spivak writes $A\subset\mathbf^n$, when he means $\mathbf^m$, leading to a mistaken understanding of what he means by a singular cube. Another important typo occurs in Thm. 2-13 (p.43), a statement later used to prove Thm. 5-1. The reader can easily come up with counterexamples to this theorem as stated. To correct this theorem, $f\circ h$ should be replaced by $f\circ h^{-1}$. Again, this is a small, but potentially bewildering error. There are numerous other smaller errors, like an unwarranted assumption in the proof of Thm. 5-2, for instance...Combined with minor typos and/or poor choice of notation, the terseness of his writing makes some parts of the text very difficult to comprehend for the beginner. The definition of the pullback of a differential form, for instance (p. 89), is given in a particularly incomprehensible way because Spivak uses the same notation in two closely related (cf. p. 77) but inconsistent ways without explanation. Long story short, Spivak implicitly used the abbreviation f^* for $(f_*)^*$ (f_* is a map defined earlier on p. 89) without any comment to that effect. Although this is a conventional abbreviation in differential geometry, this notation is baffling for the beginner if not properly explained.The errors/problems I mentioned here led to many hours of confusion. There are probably many others that I just glossed over without even recognizing any problem. (Indeed, there are several lists of errata online, as well as discussions on mathstackexchange regarding these and other errors or sources of confusion.)2) Spivak spends too long on the less interesting minutiae of derivatives and multiple integrals in chapters 2 and 3 (for example, his long discussion on the subtleties of Fubini’s theorem), and still gets some of it wrong or imprecise (like the proof of the Riemann-Lebesgue criterion for Riemann integrability, for example).3) In chapters 4 and 5, precious little is given in the way of intuition for the endless list of definitions relating to conceptually novel ideas like forms, tangent spaces, n-cubes, chains, and manifolds/diffeomorphisms. Perversely, he draws a star to define "star-like set" for the statement of Poincare lemma - a waste of space better spent on virtually anything else. Finally, the proof of the Stokes theorem gives no insight and consists of nothing more than two pages of the formal manipulation of symbols.Despite the claim in the book’s preface that only calculus and linear algebra are required as prerequisites, the truth is that mathematical maturity and sophistication, at least at the level of Chapters 1-8 of Rudin’s "Principles of Mathematical Analysis", if not a bit more, are needed for the reader to really grasp the finer points of the text. A theory course in linear algebra, e.g. one based on Halmos’ "Finite-Dimensional Vector Spaces" or Hoffman and Kunze’s "Linear Algebra", is also needed. For instance, Spivak assumes familiarity of the reader with the concept of algebraic dual spaces. The opaqueness of this book stem in part from the author’s usage of an odd mixture of elementary and advanced terminology and techniques. Thus despite his efforts, to understand what Spivak has written with any depth requires some familiarity with the methods of differential geometry. But of course, the whole point of the book’s later chapters was to familiarize undergraduate students with basic ideas from differential geometry! Thus, this book is a nightmare for self-directed learning -- a good professor or a friend who has done well in a modern differential geometry course (covering differential forms and manifolds) is essential. I suspect that a good number of the positive reviews for this book come from people who never managed to learn anything from Chapters 4 or 5.Munkres’s "Analysis on Manifolds" treats similar topics in a slightly more concrete manner. The biggest difference is that he uses the submanifolds approach rather than (singular) cubes and chains which, at this level, are not particularly useful or essential anyway and only add to the confusion. It’s more than twice as long as Spivak, which reflects a more accurate gauging of how much explanation is required for these difficult topics. It does occasionally err on the side of being too pedantic, his proofs tending to be long and wordy, and his exercises are on the easy side, but the work as a whole is orders of magnitude better than Spivak for teaching intuition. Highly recommended for the serious undergrad!Lastly, it’s worth mentioning that the contents of this little book serve as a teaser for the first volume of Spivak’s five-volume Great American Differential Geometry Book "A Comprehensive Introduction to Differential Geometry". This volume gives a thorough, clear, and engaging exposition of the ideas presented in "Calculus on Manifolds", plus a lot more. However, students might (at least initially) find the amount of material covered to be overwhelming. In this connection, Loring Tu's book "An Introduction to Manifolds" is a much more focused, less encyclopedic work geared towards beginning graduate students and is an excellent alternative. John Lee's "Introduction to Smooth Manifolds" is more complete in its coverage, but is correspondingly less accessible. Although Spivak suggests "Calculus on Manifolds" as a prerequisite for his subsequent tome, just about everything in the differential geometry portions of Calculus on Manifolds (chapters 4 and 5) reappears in it and is explained with greater clarity there.In summary, "Calculus on Manifolds" is a book of historical interest and reading it is part of becoming immersed in the "culture" of mathematics. Furthermore, the ideas that appear in "Calculus on Manifolds" form the nucleus of the modern mathematician's conception of differentiable manifolds. Thus the serious student of geometry, topology, and/or analysis would do well to grok them! However, they are better explained in nearly every other textbook and one should not hesitate to use alternate sources to learn them. If you're an undergrad (I first encountered this book as a freshman), read it for fun, and if you find these ideas fascinating, try pursuing them seriously using Munkres or, if you're up for a challenge, any of the three graduate texts I mentioned earlier. If you're a grad student, revisit "Calculus on Manifolds" after finishing a real differential geometry course to appreciate Spivak's youthful efforts to make the subject accessible to mathematical neophytes.

Let me start by saying that I think this book is the best for an advanced undergraduate or graduate student who wants to learn multivariable analysis and get an introduction to manifolds. There are several reasons for this. The first thing I think this book does well is that it has interesting problems. Unlike other competitors (i.e. Munkres), who offer no interesting problems in many sections, this book is absolutely loaded with great problems. One of the problems is even called "A first course in complex variables." Let that tell you about the quality of exercises.Another thing I like about this book is that it swiftly builds up the multivariable analysis theory without too many pit stops. One thing I hated about Munkres is that he too way too long to develop the multivariable riemann integral. Munkres takes three steps to developing it (rectangles, Jordan-measurable sets, and then open sets), and on each stage he reproves all of the facts that we know the integral should have. Spivak, on the other hand, develops the integral over rectangles, tells you in a sentence how to generalize it to Jordan-measurable sets (that's all that was needed), and then uses partitions of unity to define the more general integral. Spivak's method is faster, gives us a good look at how partitions of unity can be used, and uses the fact that the reader should be able to prove and predict the properties that the integral should have based on the assumption that we've dealt with the single variable case before. The makes Spivak a much quicker and interesting read than any other book on the subject.While I do like this book, it is not without flaws. The general opinion is that this book is a little too terse on explanations sometimes. For example, the one example Spivak gives on how to take a derivative, he identifies that derivative of the projection mapping with the i-th standard basis element. That is, he is identifying the dual space of R^n with R^n itself, all the while not telling us. While this is a nice trick that can help us take derivatives faster, this should have been mentioned in the text. Chapter 4 is rough as well. Many times Spivak will say that a theorem is obvious in an easy case, then give you a sentence on how to generalize it to the more general case. There have been many times where I have had to write my own proofs because his were lacking in detail. My margins are full of notes and missing steps because of this. For me, this wasn't too bad because I learned the material really well by being so involved, but I can easily imagine many readers being left in the dust. However, this is a good book, and these flaws only detract one star in my opinion.The next thing we need to ask is what do you need to read this. You will need a very solid understanding of single variable analysis. If you haven't read Rudin'sÃ Â Principles of Mathematical Analysis, Third EditionÃ Â yet, now is a great time to get yourself a copy. Also, you will need a strong linear algebra background. My personal favorite is Axler'sÃ Â Linear Algebra Done Right, but many people like the book by Hoffman and Kunze. One you have this background, this is the best place to go if you're looking for a quick lesson in multivariable analysis and your first introduction to manifolds.After you're done with this book, you're going to have to buy a more serious book on manifolds. This book if good for getting your feet wet, but there are so many essential things left out. I prefer the books by John Lee over Spivak's Vol. I, and so I recommend you look into those.

Everything good to say about the content of this book is referred to those with positive reviews. I, however, would like to comment on the production quality of this book. It's horrible. The text is fine, but the diagrams look like they were done by copy machine. I have an older edition to compare with, and the difference is drastic. The diagrams with annotations are useless because it's not possible to see the labels or dotted lines due to such high contrast. So, basically you're left with renditions that lose many of the features the author is trying to illustrate.

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